Homework 3

Due: 6 October, 2026

Each problem is graded on the coarse scale of $\checkmark^+$, $\checkmark$, $\checkmark^-$ and no $\checkmark$. It is also assigned a multiplier, denoting the relative importance of the problem. Both correctness and presentation are grading criteria.

Please read and make sure you understand the collaboration policy on the course missive. Extra credit problems are clearly marked below (see course missive for the details of grade calculations).

Remember to prove all your (non-elementary and not shown in class) mathematical claims, unless stated otherwise.

Each group of students should submit only 1 pdf to the corresponding Canvas assignment.

Problem 1

1. (1 $\checkmark$) Suppose we are running a property testing algorithm in a model where each query returns only a single bit. Show that, if there exists an adaptive tester using q queries, then there exists a non-adaptive tester using $O(2^q)$ queries.

2. Now consider the dense graph model, where we query the *adjacency matrix*. We will show that, for a *symmetric* property which does not depend on the vertex names/identities, we can do a lot better than an exponential slowdown when we give up adaptivity.

   (a) (2 $\checkmark$s) Suppose we have an adaptive tester using q queries, for a symmetric property. Show that there is another adaptive tester using $O(q^2)$ queries, with the following special property: whenever it queries a pair (potential edge) involving an unseen vertex (or two), it immediately queries all pairs involving the unseen vertex and all the seen ones. Furthermore, this new adaptive tester should see exactly $2q$ many vertices.

   (b) (1 $\checkmark$) Show that the $2q$ vertices might as well be chosen uniformly randomly and non-adaptively. (Hint: refer to Homework 1) Thus, there is a non-adaptive tester using $O(q^2)$ queries.

   (Side note: Goldreich and Wigderson [ECCC TR20-160] showed that the quadratic gap is in fact *tight* up to polylog factors.)

Answer one of Problems 2 and 3. Problem 3 gives less help.

Problem 2

A graph G is *Eulerian* if and only if there exists a path such that every edge is on the path exactly once. It is a well-known fact that G is Eulerian if and only if it is connected and either 1) it has all even degree vertices or 2) it has exactly 2 odd degree vertices.

The following is a structural property concerning ϵ-farness (in the bounded degree model) from being Eulerian.

Proposition 1. *Suppose G is a n-vertex graph that is ϵ-far from being Eulerian in the bounded degree model, with the degree upper bound being the constant d. Then, it has $\geq \frac{\epsilon dn}{20}$ connected components or it has $\geq \frac{\epsilon dn}{20}$ odd degree vertices.*

1. (1 ✓s) Using the above proposition, design and analyse an ϵ-tester for the Eulerian property with query and time complexity independent of n.
2. (2 ✓s) Prove the above proposition.

Problem 3

(3 ✓s)

Design and analyze an ϵ-tester for the property of being a tree in the bounded degree model, with query and time complexity independent of n.

Problem 4

(5 ✓s total)

Recall the notation that $\mathsf{PCP}[r(n), q(n)]$ denotes the class of languages with probabilistically checkable proof systems that have non-adaptive verifiers using $r(n)$ bits of randomness and $q(n)$ bits of queries, with completeness probability 1 and soundness probability $\frac{1}{2}$.

For each of the following, state and prove what complexity class it denotes (you don't need to have taken a complexity theory course for this, you have seen all these classes already).

Wikipedia has the answer to all this, but try solving this on your own! Also remember that you have to prove your answers, which Wikipedia doesn't do for you.

1. $\mathsf{PCP}[0, 0]$
2. $\mathsf{PCP}[O(\log n), 0]$
3. $\mathsf{PCP}[0, O(\log n)]$
4. $\mathsf{PCP}[O(\log n), O(1)]$
5. $\mathsf{PCP}[O(\log n), O(\log n)]$
6. $\mathsf{PCP}[0, \mathsf{poly}(n)]$
7. $\mathsf{PCP}[O(\log n), \mathsf{poly}(n)]$